Assess Sleep Stage via Jump-Diffusion Processes

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Abstract

Objective

To identify sleep stages by using EEG signals (O1A2 and O2A1) recorded during sleep.

Method

Different features are extracted from the EEG signals by the Synchrosqueezing transform. The dynamical features underlying EEG are modeled by a system of jump-diffusion processes, whose quadratic variation was used to compute the Mahalanobis-like distance between features extracted from two different time slots. The similarity level among features is evaluated by the diffusion distance and map. Instead of using the kernel support vector machine (SVM) or the K-nearest neighbors algorithm (KNN) to do the classification, the hidden Markov model (HMM) is applied to predict the sleep stage based on the alternating diffusion map of features. To prevent over-fitting, we randomly (25 times) partition the data into the training dataset (80%) and the connected validation dataset (20%) and report the averaged result.

Results

30 subjects were recruited to test the performance of the algorithm. 15 features were extracted from each 5-second window. The dynamic of features is successfully modeled by jump-diffusion processes. The result is comparable to human expert classification. The classification of awake, REM, N1, N2 and N3 sleep stages based on HMM (resp. SVM) has the overall accuracy 74% (resp. 65%). The Mann-Whitney test confirms that the performance of HMM classification is better than that of the kernel SVM.

Conclusion

This study demonstrates the capability of using EEG signals to discern sleep stages automatically.

References

[1] H.-T. Wu, R. Talmon, Y.-L. Lo, Assess Sleep Stage by Modern Signal Processing Techniques, IEEE TBME, 2015.

[2] R. R. Lederman, R. Talmon, H.-T. Wu, Y.-L. Lo, R. R. Coifman, Alternating diffusion for common manifold learning with application to sleep stage assessment, ICCASP, 2015.

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Model

Given a signal **O1A2** (or **O2A1**), let $\mathbf{y}(t) = [y^1(t)\ y^2(t)\ \cdots\ y^m(t)]$, $t \in [0,T]$ (sec) be the extracted features, which is assumed to be controlled by an underlying process θ and an unknown function f

$$\mathbf{y}(t) = f(\theta^1(t), \theta^2(t), ..., \theta^d(t)),$$

The components $\theta^1, \dots, \theta^d$ are assumed to satisfy

$$\theta^i(t) = \int_0^t a^i(\theta(s))ds + W^i(t) + J^i(t),\ t \in [0,T],$$

where a^i is an unknown drift, $W^i(t)$ is a standard Brownian motion and $J^i(t)$ is a pure jump process. The self-product $J_f^{\mathrm{T}} J_f$ of the Jacobian matrix of f can be estimated by the quadratic variation process $[\mathbf{y},\mathbf{y}](\cdot)$ of $\mathbf{y}$ through

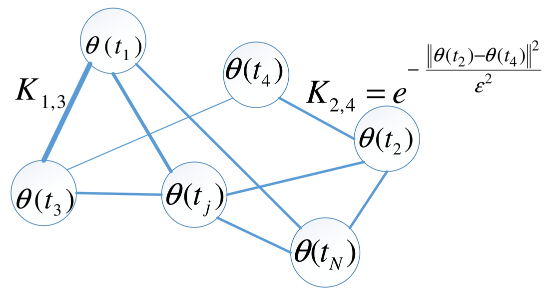
$$[y_j, y_k](t) = \int_0^t \left\{ [J_f(\theta(s))]^{\mathrm{T}} [J_f(\theta(s))] \right\}_{j,k} ds + \sum_{0<s\le t} \left[y_j(s) - y_j(s-)\right]\left[y_k(s) - y_k(s-)\right].$$

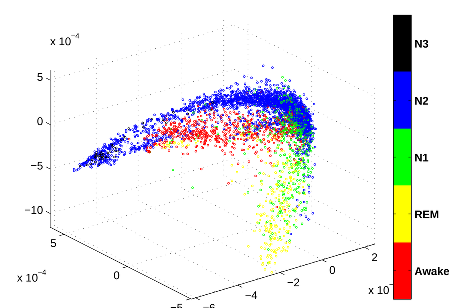
After knowing how to deal with the jump-related term,

$$\|\theta(t) - \theta(s)\|^2_{\mathbb{R}^d} \approx \frac{1}{2}[\mathbf{y}(t) - \mathbf{y}(s)][J_f(\theta(t))^{\mathrm{T}} J_f(\theta(t))]^{-1}[\mathbf{y}(t) - \mathbf{y}(s)]^{\mathrm{T}} + \frac{1}{2}[\mathbf{y}(t) - \mathbf{y}(s)][J_f(\theta(s))^{\mathrm{T}} J_f(\theta(s))]^{-1}[\mathbf{y}(t) - \mathbf{y}(s)]^{\mathrm{T}}.$$

Diffusion map Φ (DM)

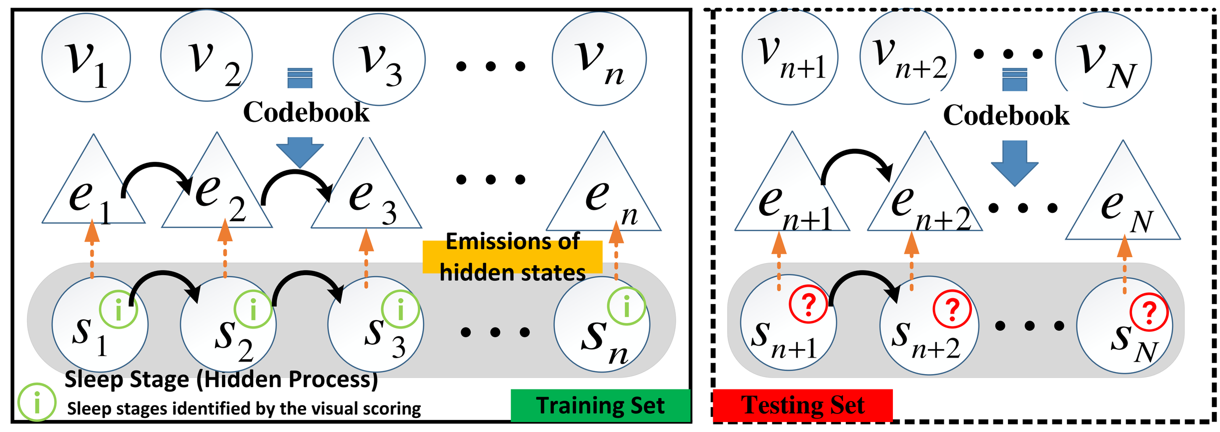
The underlying factors $\{\theta(t_i)\}_{i=1}^N \subset \mathbb{R}^d$ are viewed as the vertices of an edge-weighted graph $\mathbf{G}$. On $\mathbf{G}$, we construct a Markov chain with transition matrix $\mathbf{P} \in \mathbb{R}^{N\times N}$ by row normalizing the affinity/similarity matrix $\mathbf{K}$: $\mathbf{P} = \mathbf{D}^{-1}\mathbf{K}$, $\mathbf{D} = \mathrm{diag}(\sum_{j=1}^N K_{1,j}, ..., \sum_{j=1}^N K_{N,j})$.





Based on eigenvectors/values of $\mathbf{P}$, the truncated DM is defined by $\theta(t_j) \xrightarrow{\Phi} \begin{bmatrix} \lambda_2 u_2(j) \\ \lambda_3 u_3(j) \\ \lambda_4 u_4(j) \end{bmatrix} =: v_j.$

Classification via HMM



- create a cookbook $\mathbf{B}$ for the vector quantization: $v_j \xrightarrow{\mathrm{VQ}} e_j = \arg\min_{c\in\mathbf{B}} |v_j - c|,\ j = 1, ..., N.$
- Use the training set to estimate the transition matrix: $\forall\ s, s' \in \{\text{Awake,REM,N1,N2,N3}\}$,

$$p(s'|s) = \left[\sum_{j=1}^{n} 1\{s_j = s, s_{j+1} = s'\}\right]\left[\sum_{\ell=1}^{n} 1\{s_\ell = s\}\right]^{-1}.$$

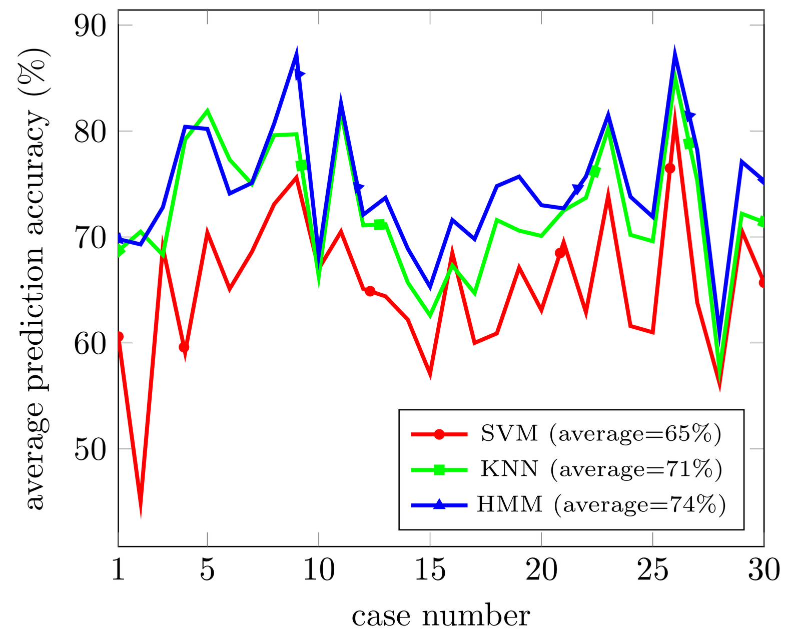
- Emission matrix (**b**). $\forall\ c \in \mathbf{B}$,

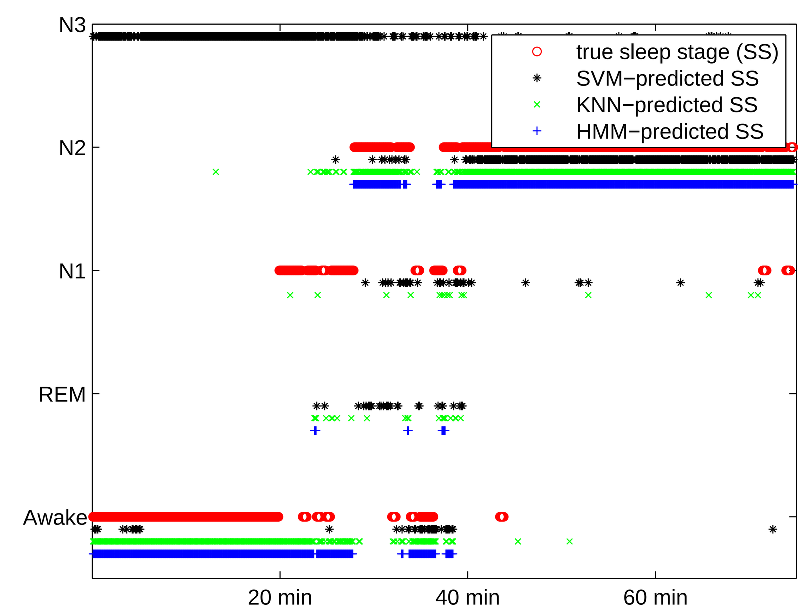
$$b_s(c) = \left[\sum_{j=1}^{n} 1\{s_j = s, e_j = c\}\right]\left[\sum_{\ell=1}^{n} 1\{s_\ell = s\}\right]^{-1}.$$

- compute and optimize the likelihood function

$$\mathrm{P}(S_{n+1} = s_{n+1}, ..., S_N = s_N | e_{n+1}, ..., e_N, S_n = s_n)$$

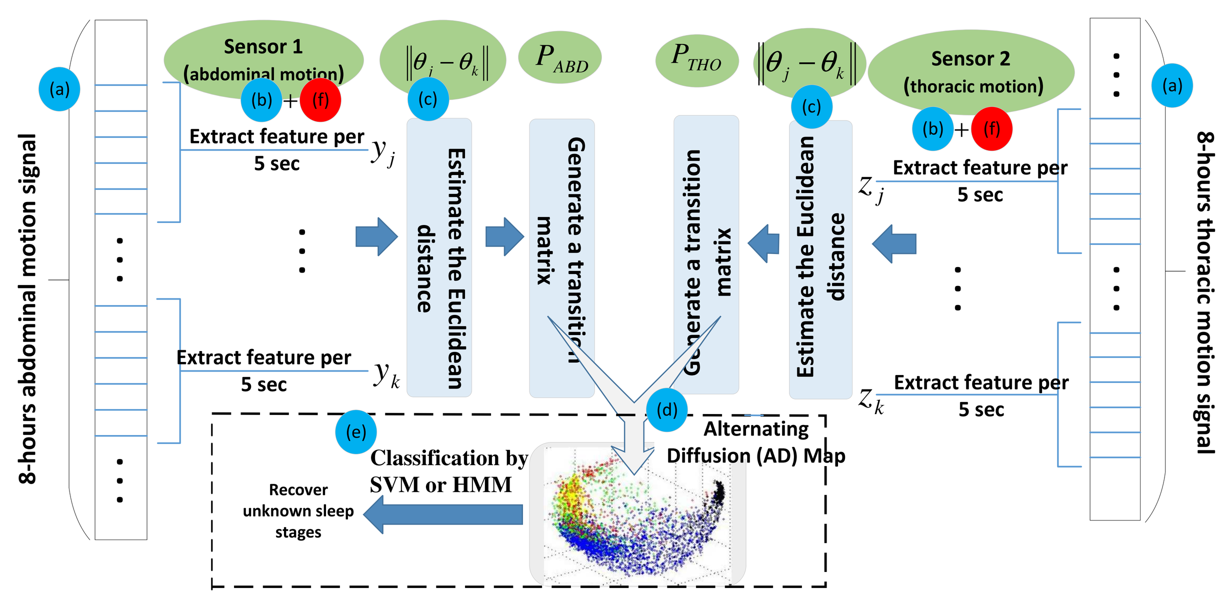
Results





- For each subject, 20% sleep period is randomly selected as the testing set.
- The accuracy of HMM (74%) is better than SVM (65%) and KNN (72%) averagely.
- The prediction of sleep stages by HMM is more *stable* than KNN and SVM.

Discussion



- For the sake of balancing between the computation complexity and number of DM points (resolution), the window size for the feature extraction is set to 5-second. The features we acquire from the EEG signals are actually finer and hidden deeply inside the signal, which may not be easily identified by human's naked eyes.
- The experimental results show that introducing jump processes to capture the normal or abnormal sleep stage transition is reasonable and it can brings positive effects on the subsequent classification work.
- More features will be taken into account. Currently, 15 features were extracted from each 5-second window. The ad-hoc method, i.e., the threshold policy, is applied to identify whether abnormal jumps occur. The corresponding performance is comparable to the model free approach [1]. The identification of jumps should be subject-dependent. More prior information is desirable for correctly identifying the occurrence of jumps.
- Our further work is to explore the possibility of realtime inter-subject sleep stage assessment and test the possibility of using this algorithm on other physiology signal like respiratory signals (THO and ABD) to discern sleep stage automatically.